Calc BC 9.2 Pg 442 #15-17, 21-23, 40

(15) $f(x) = x^3 - 2x + 4$

$f'(x) = 3x^2 - 2$

$f''(x) = 6x$

$f'''(x) = 6$

a) $f(x)$ is it's own Taylor Polynomial at $x=0$

b) $f(x) = 3 + (x-1) + \frac{6(x-1)^2}{2!} + \frac{6(x-1)^3}{3!}$

$= 3 + (x-1) + 3(x-1)^2 + (x-1)^3$

(16) $f(x) = 2x^3 + x^2 + 3x - 2$

$f'(x) = 6x^2 + 2x + 3$

$f''(x) = 12x + 2$

$f'''(x) = 12$

a) $f(x)$ is it's own Taylor Polynomial

b)

$P_3 = -2 + \frac{11}{1!}(x-1) + \frac{14}{2!}(x-1)^2 + \frac{12}{3!}(x-1)^3$

$= -2 + 11(x-1) + 7(x-1)^2 + 2(x-1)^3$

(17) $f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2$

$f'''(x) = 24x$

a) $f(x)$ is it's own Taylor Polynomial

b)

$P_3 = 1 + \frac{4}{1!}(x-1) + \frac{12}{2!}(x-1)^2 + \frac{24}{3!}(x-1)^3$

$= 1 + 4(x-1) + 6(x-1)^2 + 4(x-1)^3$

(21) $f(x) = \sqrt{x} = x^{\frac{1}{2}}$, $a = 4$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}} = -\frac{1}{4\sqrt{x^3}}$

$f'''(x) = \frac{3}{8}x^{-5/2}$

$P_0 = 2$

$P_1 = 2 + \frac{1}{4 \cdot 1!}(x-4)$

$P_2 = 2 + \frac{1}{4}(x-4) - \frac{1}{32 \cdot 2!}(x-4)^2$

$P_3 = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3$

(22)

a) $P_3 = 4 + \frac{5}{1!}x - \frac{8}{2!}x^2 + \frac{6}{3!}x^3$

$= 4 + 5x - 4x^2 + x^3$

$f(0.2) \approx P_3(0.2) = 4.848$

b) $f'(x) = P_3'(x) = 5 - 8x + 3x^2$

$f'(0.2) \approx 3.52$

(23)

a) $P_3 = 4 - (x-1) + \frac{3}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$

$f(1.2) \approx P_3(1.2) = 3.8626$

b) $f'(x) \approx P_3'(x) = -1 + 3(x-1) + (x-1)^2$

$f'(1.2) \approx -0.36$

(40) (A) $\frac{3^4}{4!} = \frac{27}{8}$